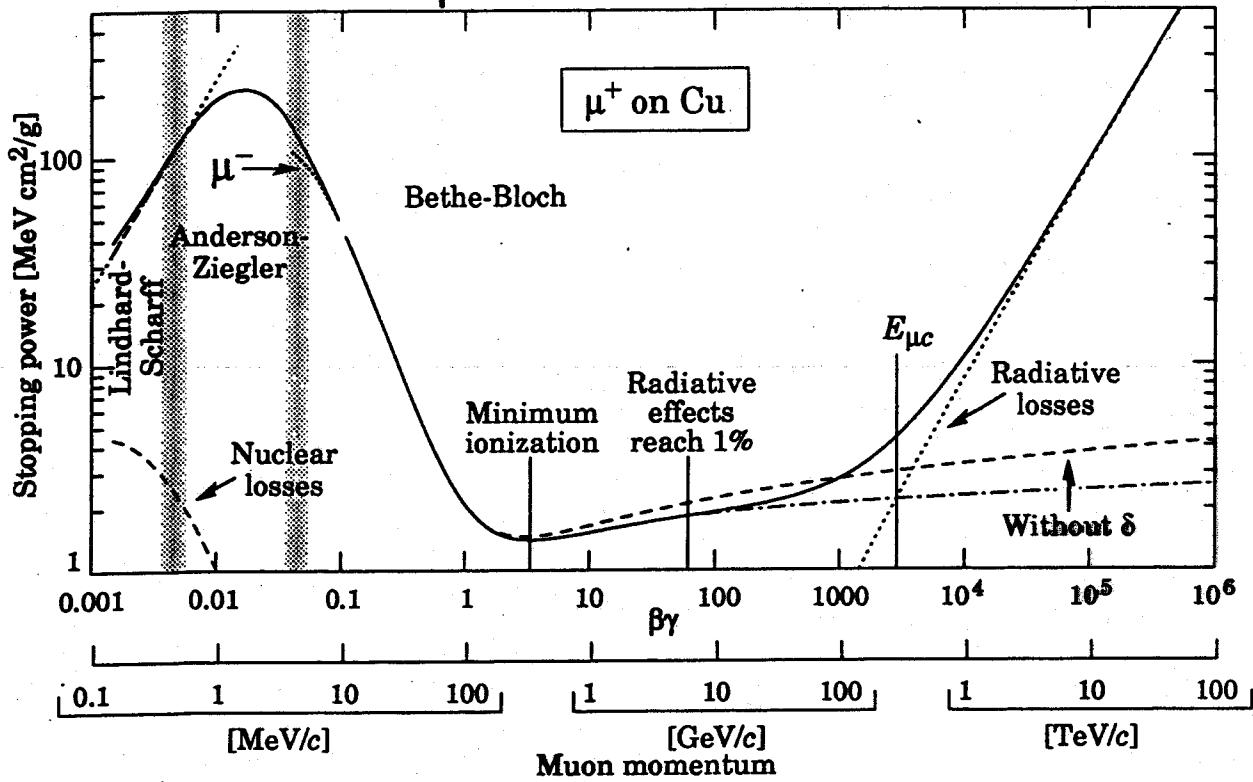
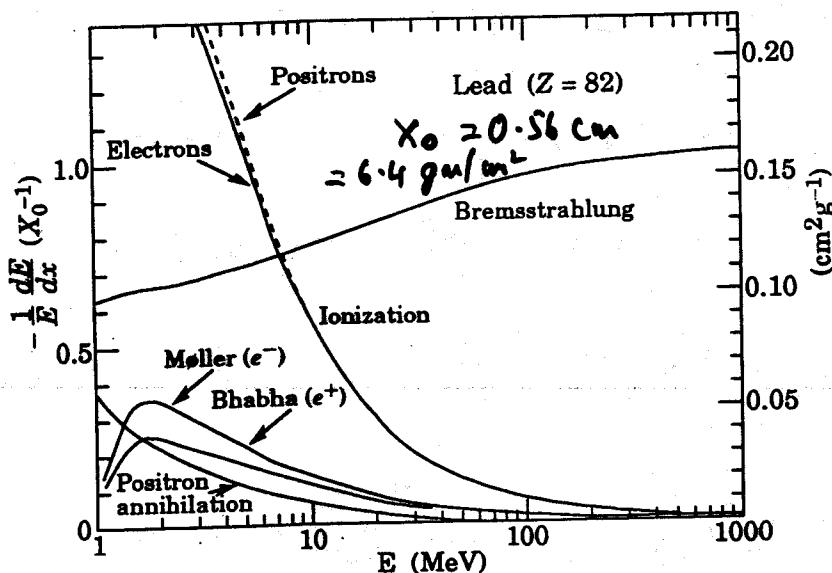


Review of Energy Loss / Stopping Power

$$-\frac{dE}{dx} = \lambda \cdot Z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2\gamma^2\beta^2}{I^2} E_{kin}^{max} - \beta^2 - \frac{\delta}{2} - \frac{c}{z} \right]$$



- Bethe - Bloch equation is valid for $\beta \gg \alpha Z$
- At low energies, $\frac{dE}{dx} \propto \frac{1}{\beta^2}$, reaches a broad minimum at $\beta \gamma \approx 4$. (Minimum Ionizing Particles)
- In light absorbers $-\left. \frac{dE}{dx} \right|_{min} = 1.5 - 2 \text{ MeV/g cm}^{-2}$
- Relativistic rise for $\gamma > 4$ ($\sim \ln \gamma$).
- Density effect reduces the relativistic rise and eventually saturates
- β dependent corrections at low β ; Radiative losses at high E



Energy where the two kinds of losses are equal is called the Critical Energy, E_c

$$-\frac{dE(E_c)}{dx} \Big|_{\text{ion}} = -\frac{dE(E_c)}{dx} \Big|_{\text{abs}}$$

Energy loss for electrons (positrons) by ionization and radiation and other mechanisms

For $Z \geq 13$,

$$E_c^e \approx \frac{550 \text{ MeV}}{Z}$$

$$E_c^\mu(\text{Pb}) = 7.4 \text{ MeV} ; E_c^\mu(\text{Fe}) = 20.7 \text{ MeV}$$

$$E_c^\mu = E_c^e \cdot \left(\frac{m_\mu}{m_e} \right)^2 ; E_c^\mu(\text{Fe}) = 890 \text{ GeV}$$

For 100 GeV μ ,

$$-\frac{dE}{dx} = \frac{E}{X_0} \cdot \left(\frac{m_e}{m_\mu} \right)^2 = 0.17 \text{ MeV/g/cm}^2$$

$$= 1.34 \text{ MeV/cm}$$

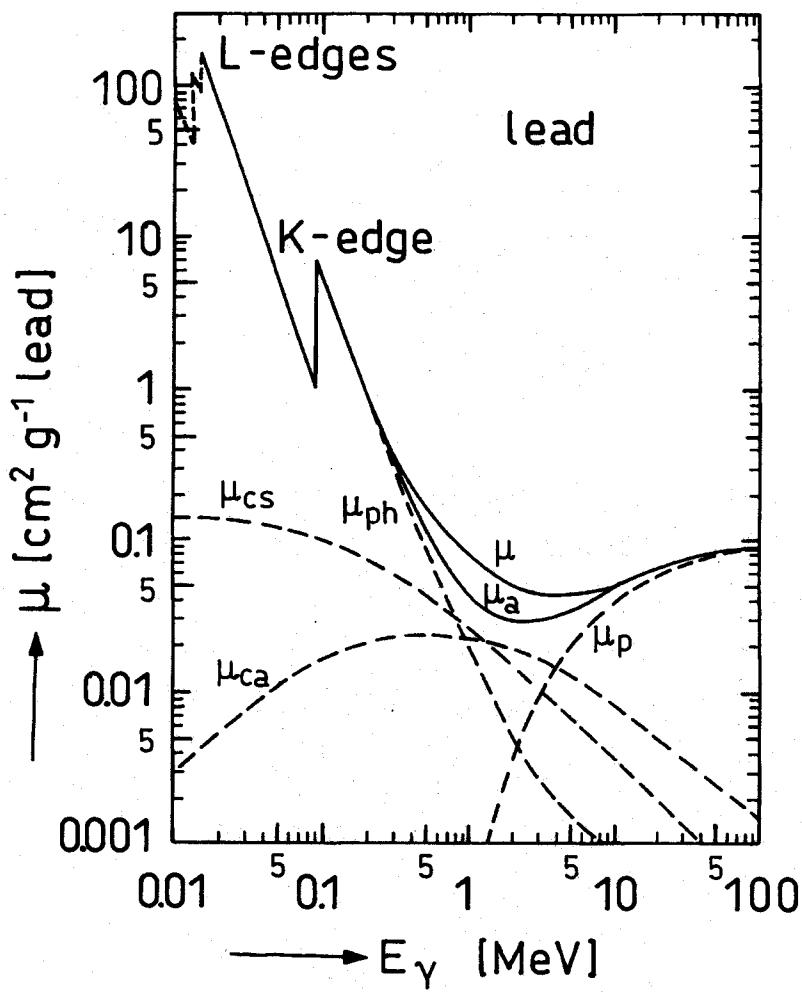


Fig. 1.14 d. Energy dependence of the mass attenuation coefficient μ and mass absorption coefficient μ_a for photons in lead [63, 73, 74, 75].

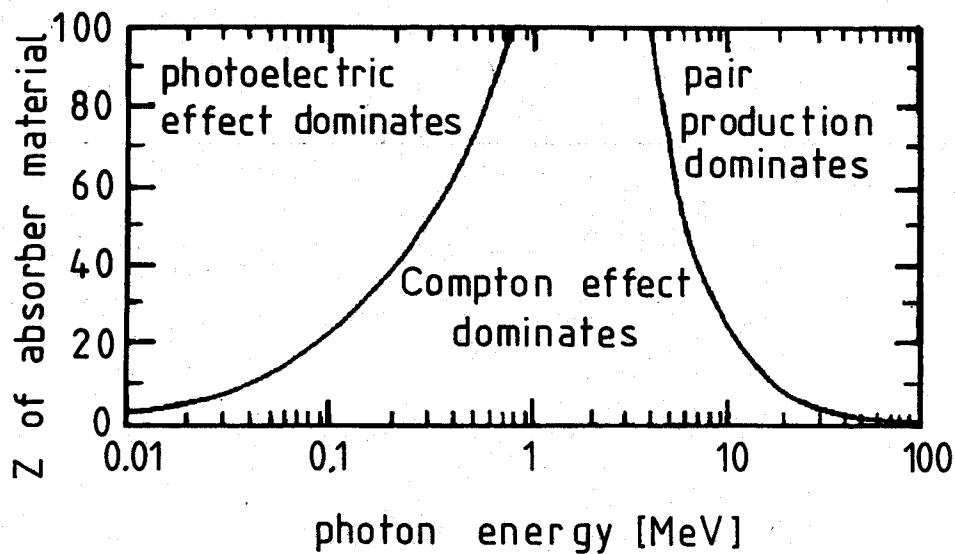


Fig. 1.15. Ranges, in which the photoelectric effect, Compton effect, and pair production dominate as a function of the photon energy and the target charge Z [37, 65, 68].

The frequency of the scattered photon:

$$\gamma' = \frac{\nu}{1 + \epsilon(1 - \cos\theta)} \quad (1) \quad \epsilon = \frac{h\nu}{mc^2}$$

The kinetic energy of the recoil electron:

$$T = mc^2 \cdot \frac{\epsilon^2(1 - \cos\theta)}{1 + \epsilon(1 - \cos\theta)} \quad (2)$$

The electron recoil angle:

$$\cos\phi = (1 + \epsilon) \left[\frac{1 - \cos\theta}{\alpha + \epsilon(\epsilon + 2)(1 - \cos\theta)} \right] \quad (3)$$

For back scattering ($\theta = \pi$), (2) becomes,

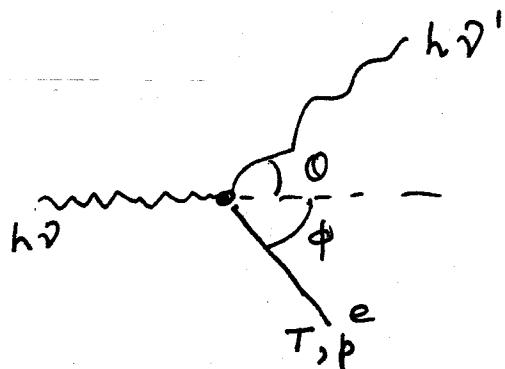
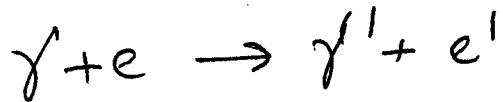
$$T = T^{\max} = \frac{2\epsilon^2}{1 + 2\epsilon} \cdot mc^2$$

For $\epsilon \gg 1$, i.e., $E_f = h\nu \gg mc^2$,

$$T^{\max} \approx E_f$$

Compton Effect

- Scattering of an incident photon off a quasi-free atomic electron.



From energy momentum conservation,

$$h\nu = h\nu' + T$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos\theta + p \cos\phi$$

$$0 = \frac{h\nu'}{c} \sin\theta - p \sin\phi$$

Using these and

$$T = \sqrt{p^2 c^2 + m_e^2 c^4} - m_e c^2,$$

One can derive a number of useful relations.

Photoelectric Effect

- First explained by Einstein in 1905.

- An interaction between photon and the atom



- Photons with energy $E_\gamma \gg E_b$, the binding energy of an electron in the atom, may be absorbed and an atomic electron ejected with kinetic energy $T = E_\gamma - E_b$.

- "Resonant" absorption occurs with E_γ near the bound state energies of the electrons,

$$E_n = -13.6 \text{ eV} \cdot \frac{Z^2}{n^2} \quad (n = \text{principal quantum number})$$

- The photoelectric cross section

$$\sigma_{pe} = \frac{32\pi}{3} \cdot \sqrt{2} \cdot Z^5 \alpha^4 \left(\frac{m}{\hbar w} \right)^{7/2} \cdot (\alpha \beta)^2$$

$$\sigma_{pe} \propto \frac{Z^5}{E_\gamma} \quad \text{for high energies}$$

- Main application in PMTs

Interactions of Photons

Photons, unlike heavy charged particles, get attenuated in matter easily.

The number of photons lost from a monoenergetic beam of N photons, while passing through a thickness dx of a material is,

$$dN = -\mu N dx ; N = N_0 e^{-\mu x}$$

where μ = mass attenuation coefficient, and is related to the probability that a photon will be scattered or absorbed in the material.

∴ The intensity of a photon varies as

$$N = N_0 e^{-\mu x}$$

i.e., decrease exponentially with thickness of the material traversed.

Note: Contrast with energy attenuation of charged particles during radiation loss

\therefore The Radiation length of a material

$$X_0 = \frac{A}{4\alpha N_A Z(Z+1) \pi e^2 \cdot \ln(183 Z^{-1/3})} \text{ g/cm}^2$$

Note: $X_0 \propto Z^{-2}$, $X_0 \propto m^2$

For a mixture or a compound

$$X_0 = \frac{1}{\sum_{i=1}^N f_i / X_0^i}; \quad f_i = \text{fraction of the } i^{\text{th}} \text{ component by weight}$$

$X_0^i = \text{Radiation length of the } i^{\text{th}} \text{ component.}$

$$-\frac{dE}{dx} = \frac{E}{X_0} \Rightarrow E = E_0 e^{-x/X_0} \quad (X_0(\text{Fe}) = 1.76 \text{ cm})$$

Exponential attenuation of energy by radiation loss

Compare with ionization loss:

$$\left. \frac{dE}{dx} \right|_{\text{ion}} \propto Z; \quad \left. \frac{dE}{dx} \right|_{\text{rad}} \propto Z^2$$

$$\left. \frac{dE}{dx} \right|_{\text{ion}} \propto \ln E; \quad \left. \frac{dE}{dx} \right|_{\text{rad}} \propto E$$

$(\gamma > 4)$

Bremsstrahlung

A charged particle accelerated or decelerated in the Coulomb field of a nucleus can emit a fraction of its energy as real photons.

The energy loss due to radiation can be calculated as :

$$-\left(\frac{dE}{dx}\right)_{\text{rad}} = N \int_0^{\gamma_0} h\nu \frac{d\sigma}{d\nu}(E, \nu) d\nu$$

$$-\left(\frac{dE}{dx}\right)_{\text{rad}} = 4\alpha N_A r_e^2 \cdot Z^2 \cdot \frac{Z^2}{A} \cdot E \cdot \ln \frac{183}{Z^{1/3}}$$

$\alpha = e^2/kc$; $r_e^2 = e^2/mc^2$; m = mass of the incident particle

To take into account interactions with electrons,

$$Z^2 \rightarrow Z^2 + Z = Z(Z+1)$$

For electrons, $Z=1$, $m=m_e$, we write

$$-\frac{dE}{dx} = \frac{E}{X_0} \Leftarrow \text{This defines "Radiation Length"}$$

Energy Loss of Electrons

Recall the Bethe-Bloch formula for heavy charged particles:

$$\frac{dE}{dx} = K \cdot Z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2/\beta^2}{I^2} \cdot E_{kin}^{max} - \beta^2 - \frac{\gamma}{2} \right]$$

~~$\frac{\gamma}{2}$~~

ignore for now

for relativistic particles, $\beta \rightarrow 1$

with $E_{kin}^{max} = 2mc^2\gamma^2\beta^2$,

$$-\frac{dE}{dx} \approx K \cdot \frac{Z}{A} \left[\ln \frac{2mc^2}{I} + 2\ln\gamma - 1 \right]$$

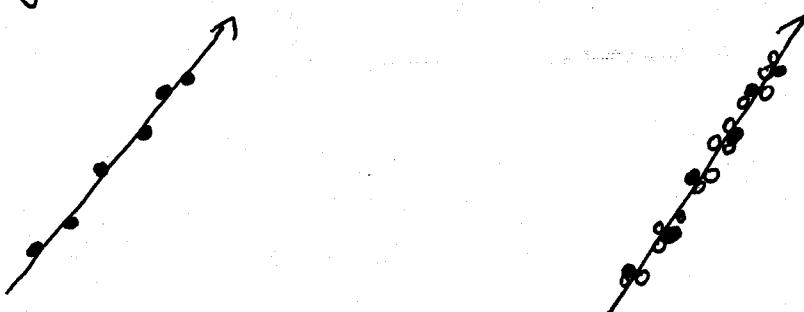
When the incident particle is an electron,
one cannot distinguish between the primary
and the secondary electron after the collision.

$$E_{kin}^{max} = \frac{1}{2}(E - mc^2) \approx \frac{1}{2}E \text{ for } E \gg mc^2$$

$$-\frac{dE}{dx} = K \cdot \frac{Z}{A} \left[\ln \frac{2mc^2}{I} + \frac{3}{2}\ln\gamma - \frac{1}{2}\ln 4 - 1 \right]$$

Ionization Yield

- Total number of electron-ion pairs produced by the passage of a charged particle.
- Primary ionization + secondary ionization



The electrons produced in primary collisions may have enough energy to ionize other atoms.

- $n_{\text{Total}} = \frac{\Delta E}{W} = \frac{\frac{dE}{dx} \cdot \Delta x}{W}$

ΔE = Total energy loss

Δx = Thickness traversed

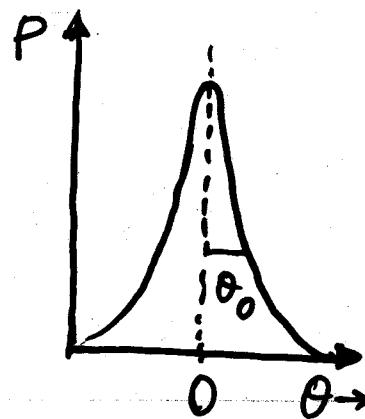
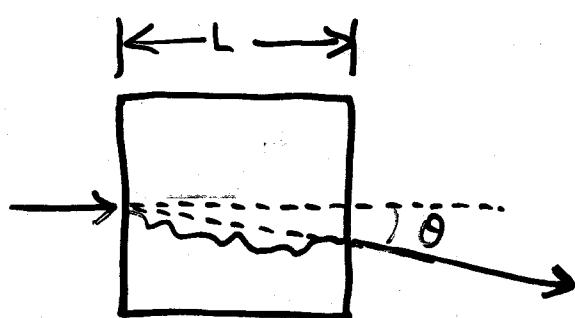
W = effective energy loss / pair

$$W \approx 26 \text{ eV (Ar)} \quad n_p = 29 \text{ fm}^{-3}, n_{\text{total}} = 94 \text{ /cm}^3$$

$W \approx 3 \text{ eV in Semiconductors}$

Multiple Coulomb Scattering

- A charged particle traversing a medium can suffer many small angle scatters in the Coulomb potentials of nuclei



$$P(\theta) = \frac{1}{\sqrt{2\pi}\theta_0} \exp\left(-\frac{\theta^2}{2\theta_0^2}\right)$$

$$\theta_0 = \sqrt{\langle \theta^2 \rangle} = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{\frac{L}{X_0}} \left[1 + 0.038 \ln \frac{L}{X_0} \right]$$

p: momentum in MeV/c , βc : velocity, z = charge of the scattered particle, X_0 = Radiation length

$$\boxed{\theta_0 \approx \frac{13.6}{\beta c p} \sqrt{\frac{L}{X_0}}}$$

Energy Loss by Photonuclear Interactions

charged particles can interact inelastically via virtual photons with nuclei and lose energy

$$-\frac{dE}{dx} \Big|_{\text{photomel}} = b_{\text{nuc}}(Z, A, E) \cdot E$$

For 100 GeV muons in Fe,

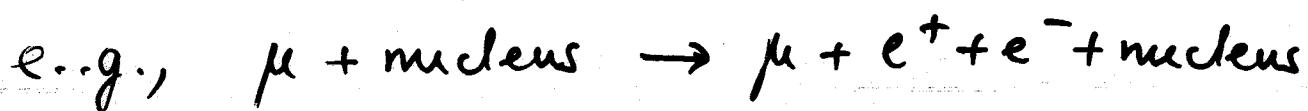
$$-\frac{dE}{dx} \Big|_{\text{photomel}} = 0.04 \frac{\text{MeV}}{\text{g/cm}^2}$$

Total Energy Loss

$$\begin{aligned} -\frac{dE}{dx} \Big|_{\text{total}} &= -\frac{dE}{dx} \Big|_{\text{ion}} + \frac{dE}{dx} \Big|_{\text{brem}} - \frac{dE}{dx} \Big|_{\text{pair}} - \frac{dE}{dx} \Big|_{\text{ph}} \\ &= \alpha(Z, A, E) + b(Z, A, E) \cdot E \end{aligned}$$

Direct Pair Production

At high energies, e^+e^- pairs can be produced by virtual photons in the Coulomb field of the nuclei.



$$-\frac{dE}{dx} \Big|_{\text{pair}} = b(Z, A, E) \cdot E$$

b varies only slightly with E

$$\therefore -\frac{dE}{dx} \Big|_{\text{pair}} \propto E$$

For 100 GeV μ , in Fe

$$-\frac{dE}{dx} \Big|_{\text{pair}} = 0.3 \frac{\text{MeV}}{\text{g/cm}^2} \quad (= 2.36 \text{ MeV})$$

